

DUALITY VERSUS SUPERSYMMETRY<sup>1</sup>TOMÁS ORTÍN<sup>2</sup>*Department of Physics, Queen Mary and Westfield College, Mile End Road, London E1 4NS, U.K.*

## ABSTRACT

We study the effect of S-duality and target-space duality transformations of  $N = 4, d = 4$  and  $N = 1, d = 10$  supersymmetric configurations on their Killing spinors. We find that, under reasonable assumptions, the dual configurations are also supersymmetric and that the Killing spinors transform in a simple way.

There is an increasing interest in finding new classical backgrounds of string theory describing black holes, gravitational waves etc. Many have been found recently as solutions of the low-energy effective action and exact CFTs, and some of the most interesting ones have unbroken supersymmetries. On the other string theory has several *duality* symmetries that relate very different-looking backgrounds. T-duality [1] relates two backgrounds with an isometry which are described by equivalent CFTs. From a different point of view T-duality also relates two solutions of the leading order in  $\alpha'$  effective action with an isometry. S-duality is essentially a symmetry of  $N = 4, d = 4$  supergravity [2] (the leading order in  $\alpha'$  of the effective action in  $d = 4$ ) formulated with a pseudoscalar axion, although it has been conjectured to be an exact non-perturbative symmetry of string theory [3]. Our aim is to study if these duality transformations preserve unbroken supersymmetries and, if they do, what is the transformation law of the Killing spinors. We will do it in the framework of the leading order effective action.

In the case of S-duality our results can be stated as follows [4]: If  $\{g_{\mu\nu}, A_\mu^I, \lambda = a + ie^{-2\phi}\}$  is a solution of  $N = 4, d = 4$  supergravity (Einstein frame) admitting the Killing spinor set  $\epsilon^I$ , and  $\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$  is an  $SL(2, Z)$ -duality transformation, then the transformed configuration  $\{g_{\mu\nu}, A_\mu^{II}, \lambda'\}$  is also a solution admitting the Killing spinor

$$\epsilon'^I = e^{\frac{i}{2} \text{Arg}(S)} \epsilon^I, \quad S = \gamma\lambda + \delta. \quad (1)$$

To prove this result there is no need to use the equations of motion and therefore it applies to more general configurations which are not solutions of the equations of

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<sup>2</sup>E-mail address: ortin@qmchep.cern.ch

motion. However, if Maxwell's equation is not satisfied by the configuration then, in general, the Bianchi identity will not be satisfied after the transformation and one can not properly speak of an  $N = 4, d = 4$  supergravity configuration. The S-convariance of the Killing spinor set implies the invariance of the Bogomolnyi bound in axion-dilaton black holes [4].

In the case of T-duality our results can be stated as follows [5]: Let  $\{\hat{g}_{\hat{\mu}\hat{\nu}}, \hat{B}_{\hat{\mu}\hat{\nu}}, \hat{\phi}\}$  be a solution of  $N = 1, d = 10$  supergravity admitting the Killing spinor  $\epsilon$ . If the fields do not depend on the coordinate  $x$  it is natural to use a Kaluza-Klein-type basis of zehnbeins  $\hat{e}_{\hat{\mu}}^{\hat{a}} = \begin{pmatrix} e_{\mu}^a & kA_{\mu} \\ 0 & k \end{pmatrix}$ . If in this basis  $\epsilon$  does not depend on  $x$ , then the T-dual configuration with respect to  $x$  admits a Killing spinor  $\tilde{\epsilon}$  where

1. if  $x$  is space-like  $\tilde{\epsilon} = \epsilon$ .
2. if  $t$  is time-like  $\tilde{\epsilon} = \gamma_x \epsilon$ .

An interesting example that illustrates this result is provided by the classes of solutions known as Supersymmetric String Waves and Generalized Fundamental Strings [6]. These two classes of solutions are related by T-duality and admit Killing spinors. In a KK-type basis of zehnbeins the Killing spinors are equal. In particular some SSW are dual to the four-dimensional extreme electric dilaton black hole uplifted to  $d = 10$  in a way in which the preservation of the four-dimensional unbroken supersymmetry is guaranteed [7].

Another example is provided by the four-dimensional extreme magnetic dilaton black hole uplifted to  $d = 10$  preserving supersymmetry [8]. In  $d = 10$  its Killing spinors are constant spinors constrained by  $(1 \pm \gamma_{1234})\epsilon_{\pm} = 0$ . The configuration is invariant under T-duality in the time direction, and the constraint on the killing spinors is invariant under multiplication by  $\gamma_0$ , confirming the above result.

These results can be extended in a number of interesting ways: inclusion of (non-Abelian) vector fields, continuous  $SO(1,2)$  duality etc and the results will be published elsewhere [5].

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